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# NONLINEAR RR LYRAE MODELS WITH TIME DEPENDENT CONVECTION

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## ABSTRACT

Results of convective, nonlinear RR Lyrae models are presented. The standard mixing length theory has been used with time dependence being introduced through the convective velocity phase lag technique. Turbulent pressure and turbulent viscosity are also included. Results are compared with those of other time dependent convection theories.

## INTRODUCTION

One of the lingering problems in stellar pulsation theory is that of time dependent convection. Whenever the time scale for convection, defined as the amount of time necessary for a convective eddy to travel one mixing length, is of the same order as the pulsation time, consideration must be given to the finite amount of time necessary for the adjustment of convection to changing conditions. Several attempts have been made recently to incorporate time dependence in nonlinear stellar models. Deupree (1979) used a two dimensional simulation of convection to investigate the red edge of the RR Lyrae instability strip and found that convection does indeed suppress pulsation at about the right location in the HR diagram. A 2D approach is, however, incompatible with existing 1D nonlinear pulsation codes. Stellingwerf (1982) proposed a 1 D nonlinear, nonlocal, time dependent convection theory based on a phase lagging of the convective velocity. His approach was then used to investigate several features of the RR Lyrae instability strip, including red and blue edges (1984a). He also described, in detail, effects of time dependent convection on a model (model 2.5) located in the center of the fundamental mode instability strip (1984b,c). Very recently, two papers have been published that consider the effect of time dependent convection on one zone models in an attempt to understand more clearly

the relationships between various convective parameters and pulsation (Pesnell, 1985; Stellingwerf, 1986).

### EQUATIONS

In the present study, modifications are made to the standard mixing length theory (Böhm-Vitense, 1958) to incorporate time dependence through a convective velocity phase lag. It is hoped that this simplified approach will yield reasonable results. The convective velocity of a zone at time step  $n$  has been modified by setting

$$v_n = v_{n-1} + \tau(v_0 - v_{n-1})$$

where

$$\tau = \Delta t * v_n / \ell$$

$\Delta t$  is the time step,  $\ell$  is the mixing length, and  $v_0$  represents the instantaneous convective velocity determined from local conditions.

Nonlocal effects have also been incorporated by weighting the current convective velocity of zone  $i$  with the convective velocities of neighboring zones from the previous time step,

$$v_n^i = a_{i-1} v_{n-1}^{i-1} + a_{i+1} v_{n-1}^{i+1} + (1 - a_{i-1} - a_{i+1}) v_n^i$$

where the weighting factor for zone  $k$  is given by

$$a_k = (1 - |r_k - r_i| / \ell) / 3$$

with  $r_k$  being the radius of zone  $k$ .

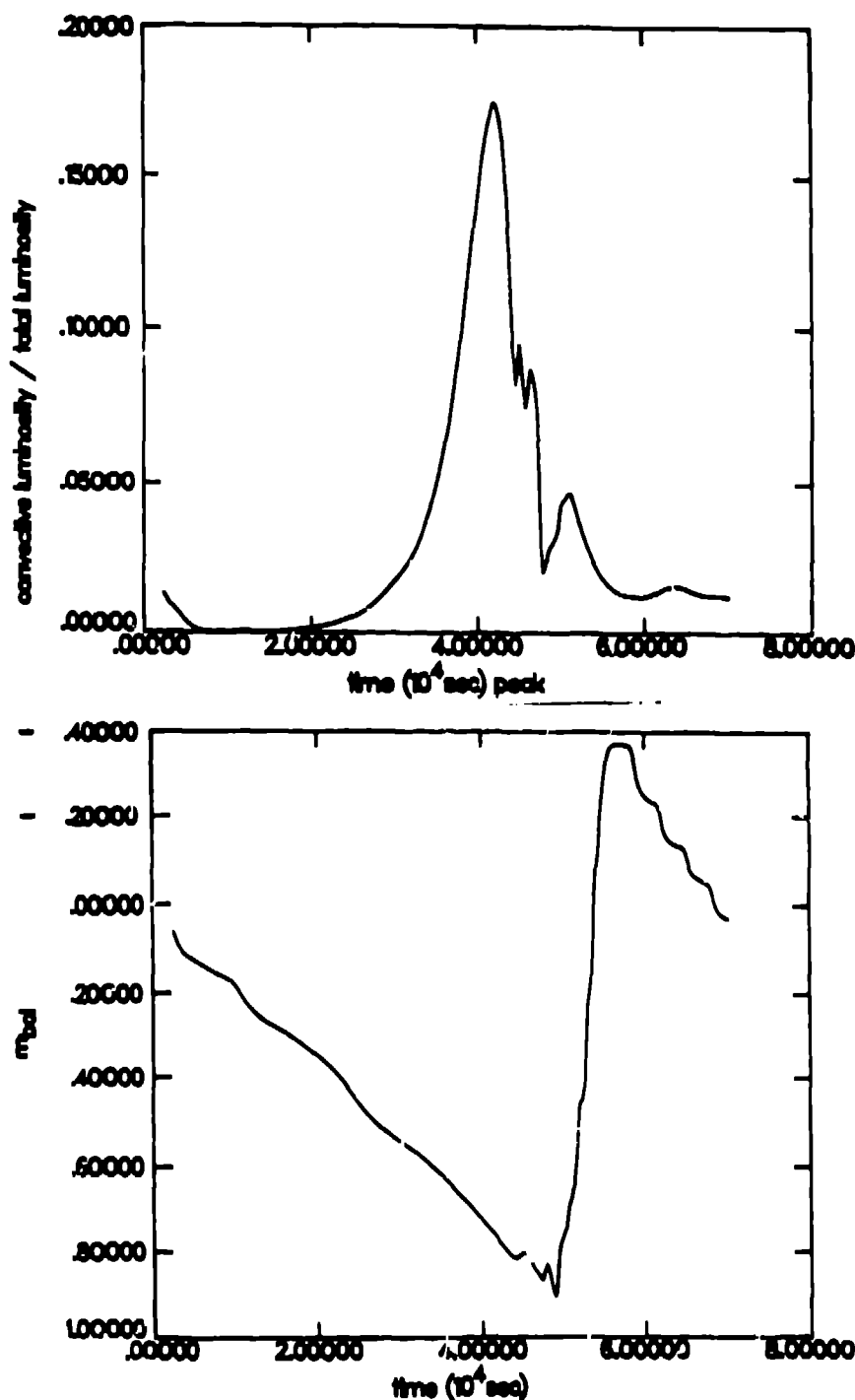
Other contributions due to the effect of convection have been included in the form of turbulent pressure, energy, and viscosity.

### RESULTS

Two 60 zones models were calculated, both with an initial fundamental mode velocity of 20 km/s. The first was model 2.5 of Stellingwerf (1984b) with  $L=63 L_\odot$ ,  $M=0.578 M_\odot$ ,  $T_{\text{eff}}=6500$  K,  $\ell/H_p=1.5$ , and  $(Y,Z)=(0.299,0.001)$ . The initial model was integrated inward to 1% of the radius using 14% of the mass. In the static model two convection zones exist, one in the hydrogen ionization region carrying 97% of the total flux and one in the helium ionization region carrying 2% of the total flux. The linear fundamental mode period is 0.812 d and the growth rate is 0.0926.

After the initial perturbation the model's amplitude grew rapidly over approximately 50 cycles to a limiting amplitude of 75 km/s and 1.2 magnitudes. During the growth to limiting amplitude the strength of the convection zones steadily decreased, with the hydrogen ionization region carrying a maximum of 18% of the flux shortly before minimum radius. The figures show both the variation in absolute bolometric magnitude and the ratio of maximum convective luminosity to total luminosity for a typical period at limiting amplitude. Maximum radius occurs at approximately  $1.4 \times 10^4$  sec. and minimum radius occurs at  $5.1 \times 10^4$  sec. It is interesting to note that Stellingwerf (1984b) finds a nearly saturated convective flux at approximately the

same phase as is found here. Not surprisingly, his limiting amplitude is significantly less than in the current study. He also finds a prominent dip in the rising branch of his light curve that is not present in our calculations.



This model was also tested for stability against other modes at limiting amplitude using Stellingwerf's (1974) periodic solution method. It was found that none of the overtones were pulsationally unstable as would be expected for an object in the center of the fundamental mode instability strip.

The second model studied here is a convective version of a model of Hodson and Cox (1982) located in the region of the double mode RR Lyrae variables. This model

has  $L=59 L_{\odot}$ ,  $M=0.65 M_{\odot}$ ,  $T_{\text{eff}}=7000 \text{ K}$ ,  $z/H_p=1.5$ , and  $(Y,Z)=(0.299,0.001)$ . The static model was integrated to 8% of the radius using 6% of the mass. The linear fundamental mode period is 0.544 d with a growth rate of 0.0094; the period ratio of the first overtone to the fundamental mode is 0.744. Due to the smaller growth rate and limited computing time, this model could not be followed long enough to determine if the higher overtones present in the initial perturbation would damp out. However, it does appear that a limiting amplitude of approximately 40 km/s can be expected. It was found that a similar phasing of the convective flux exists between this model and the previous case even though only 4% of the total flux is ever carried by convection. This small amount does seem to be sufficient to give a smaller amplitude than the 52 km/s obtained in the purely radiative model of Hodson and Cox.

#### CONCLUSIONS

The results obtained here for a modified version of the standard mixing length theory do seem to give reasonable results for the models calculated. Apparently the presence of pulsation tends to decrease the convective flux, with maximum flux occurring during the compression phase, just at the time when the radiative flux is at a minimum, thus limiting the final amplitude. Work still remains to determine if enough damping will exist to stop pulsation completely at the red edge of the instability strip.

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